Determining the Impact of the Maintenance and Salvage Cost's Function Parameters on an Equipment Optimal Replacement Time Computation Under the Economic Life Cycle Strategy: Case Study of a Commuter Bus

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Abstract

In the business of fleet management, operators or owners are faced with the difficult task of chosen between a fixed-time based replacement policy and a breakdown replacement strategy, in which a vehicle is replaced only when the repairs become nearly impossible. The focus in the choice made is whether it will attract a long- run cost effectiveness, as a fixed time scheduled if not properly modeled could result to underutilization of a vehicles, leading to profit loss or unnecessarily incurred high maintenance cost due to out of useful life cycle usage. In this study, a logarithmic transformation of costs data sets of an economic life cycle model is carried out to introduce the needed linearity in a vehicle age maintenance and salvage cost distribution for the determination of the respective costs function parameters required for a more accurate optimal replacement time estimation.

The cost comparison of the improved model due to its data transformation and that of the untransformed data is done alongside their optimal replacement time results, to ascertain the sensitivity of the costs function parameters to the optimal replacement time.

While the model based on transformed data reported a reasonable replacement time of 18 months replacement period in ten years planning horizon, that of the untransformed that reported an unrealistic optimal time that is intractable. Estimated costs and actual costs for the transformed data-based model showed little variations of about 0-150% for the maintenance cost which suffered greater disparity in the sourced data distribution and 0-109% for the salvage cycle costs. The model based on untransformed data set recorded wide variation in terms of 1000% increment in the estimated value over the actual recorded costs.

Key Words: *Logarithmic: Linearity: Life – Cycle: Optimal Time: Salvage Costs: Operational Costs*

1. INTRODUCTION

Optimal replacement time determination in fleet management is an imperative, required to ensure the minimization of ownership cost, through adequate balancing of maintenance and salvage costs.

The decision of when to replace a commercial vehicle is a complex challenge faced by fleet operators, transportation companies, and logistics managers. Over time, commercial vehicles experience wear and tear, leading to increased maintenance costs, fuel inefficiencies, and higher risks of unexpected breakdowns. If a vehicle is replaced too early, significant financial resources may be wasted due to under-utilization, while delaying replacement can result in excessive operational costs and service disruptions (Johnson & Walker, 2021). Striking a balance between cost efficiency and vehicle reliability is crucial, yet existing approaches to vehicle replacement lack a systematic and data-driven model that integrates financial, technical, and operational factors into decision-making. The absence of a comprehensive framework leaves fleet managers relying on heuristic methods, which often result in sub-optimal financial and operational outcomes.

Most commercial vehicle replacement decisions are traditionally based on fixed schedules, where vehicles are replaced after a predetermined number of years or mileage thresholds (Brown et al., 2020). However, this approach does not account for real-time variations in vehicle performance, usage intensity, or maintenance costs. Additionally, reactive replacement strategies, in which vehicles are replaced only after experiencing major breakdowns, often lead to increased downtime, reduced productivity, and higher unexpected repair costs (Anderson & Clark, 2022). The lack of a predictive and cost-optimized replacement model means that many organizations struggle to determine the best time to retire and replace their vehicles, leading to financial inefficiencies and reduced service quality.

Most fleet owners make the uninformed decision of continuously spending on maintenance costs beyond necessary even outside the useful life cycle of a vehicle with erroneous impression that, it is more economical than replacement, as long as the existing vehicle can offer gains on services.

Existing replacement models, attempt to proffer solution to this misleading business position by either determining the optimal time or usage in which overall ownership costs is lowest. The objective is the Minimization of life cycle sum of maintenance and replacement costs with recourse to age, mileage, resale value and cost of new equipment and focusing on whether to retain or replace the existing vehicle, considering operation costs and current health status (operational condition), or the determination of the thresholds for which repair costs exceeds replacement costs, taking cognizance of reliability, safety and technological advancement factors, fan et al (2001).

The traditional equipment optimal replacement models involving average life-cycle cost minimization under replacement, maintenance and salvage costs are very cumbersome to solve for the optimal timing, due to the resulting transcendental equation. The complexity arises from the included exponential terms and functions parameters in the resulting equation, hence requiring a graphical solution. The functions parameters are determined from initial values of the maintenance and salvage costs at time t=0, with subsequent equipment age salvage and maintenance costs being a percentage increment of the initial value for the maintenance cost and a proportional decrease in the initial value for the salvage cost, as though the increase or decrease is uniformly distributed within the ages to constitute a uniform proportionate costing trend. In practical condition, the maintenance and salvage cost may not increase or decrease with the same proportion across the service duration, owing to factors such as, operators varying expertise for non-permanent operators, different service period environmental impact due to seasonality, changes in government economic policies as it relates to taxes and duties, exchange rate and inflation effects, and deplorable road condition for transport equipment. Perhaps these factors may favour high maintenance and replacement cost in some periods and vice versa. For periods with high maintenance and replacement costs, prospective owners of vehicles may prefer already used cars allowing for high salvage value with insignificant depreciation rate due to regular maintenance. In other cycles where the aforementioned factors reflect low replacement and maintenance costs, even with appreciable salvage value due to affordable maintenance cost, replacement may be more preferred. Should there be the occurrence of these two extremes of the operating environment within a planning horizon, incurred maintenance cost and selvage values recorded within the horizon will experience a non-uniform distribution. In this study, to accommodate this irregularity resulting in non-uniformity in the variation of the costs data sets, a regression and logarithmic transformation analysis of the costs data set is done to see which is more potent and of better results for the economic life cycle optimal replacement time strategy, here referred to as the conventional approach. The solution of the conventional method through the determination of the maintenance and salvage cost's function parameters with the untransformed data sets is also examined to underscore the sensitivity of errors in the function parameters to the determination of the optimal replacement time

2. MATERIAL AND METHODS

The materials and methods required and deployed in this study are presented in the format below.

2.1 MATERIALS

The materials required in the study includes the following:

1. Published materials on optimal equipment replacement strategies and regression analysis.
2. Sourced secondary data on the operations of a commuter bus.

2.2 METHODS

A theoretical (mathematical modelling) and secondary data research method are adopted in this study to improve the existing economic life cycle cost model and relate its result's differentials under varied data form.

2.3 MODEL ANALYSIS.

Consider a commuter bus with a constant replacement cost (**M**), such that at any instance, its maintenance cost rate at age **n** years is C (n), and the salvage value at age **n** is S(n). suppose at some times (t) within a planning horizon, these costs vary, then there exists an optimal time (t) to replace the bus for which the total cost per unit time G (t) will be minimal, where G (t) is the ownership or operating cost per unit time. This cost G (t), is the sum of the replacement and maintenance costs minus the salvage cos. The ownership cost per unit time is reported in Nahumas (2009) as follows;

$$G(t) = M + \int_0^t C(\mathrm{n})\ dn - \mathrm{S(t)} ------------ (1)$$

Note, here that the time (t) represent a replacement cycle, where a replacement cycle is defined as the time between successive replacements. Hence G(t) represents the total ownership or operating cost per replacement cycle. Subsequently, $\int_0^t C(n)\,dn,$ represents the maintenance cost rate.

The decision variable of equation (1) is the time between the introduction of the bus into operation up until it is replaced, hence the objective is to determine the optimal cycle time that minimizes the average cost per unit time, where the average cycle cost is given as;

$$G(t) = M/t + \frac{1}{t}\int_0^t C(n)\,dn - \frac{S(n)}{t} -------------- (2)$$

Here, it is assumed that the replacement cycles are identical, and that in what follows, the equipment(bus)age maintenance and salvage costs are uniformly distributed to warrant for same proportionate representation.

Finding the optimal time t^* for equation (2) requires that,

$$G(t^*) = -\frac{M}{t^2} + \frac{\mathrm{B(t)}}{t^2} + \frac{\mathrm{C(t)}}{t^2} + \frac{\mathrm{S(t)}}{t^2} - \frac{\mathrm{S(t)}}{t^2} = 0$$

$$\Rightarrow \mathrm{t(t)} + \mathrm{S(t)} = \mathrm{M} + \mathrm{B(t)} + \mathrm{tS(t)} -------------------- (3)$$

$$\text{where } \mathrm{B(t)} = \int_0^t \mathrm{C(n)\,dn} ------------------------ (4)$$

As reported in several literature including Nahmias (2009), the exponential function best describes the increasing and decreasing maintenance and salvage cost respectively of an equipment in operation over time, hence we assume the following exponential cost functions;

$\mathrm{C(n)} = \alpha e^{\beta n}$------------------------- (5)
Where $\alpha , \beta > 0$
And
$\mathrm{S(n)} = \gamma e^{-\phi n}$ -------------------- (6)
Where $\gamma , \phi > 0$
Hence equation (3) after substituting eqn. (5) and (6) can be rewritten as follows;
$t\, \alpha e^{\beta t} + \gamma\, e^{-\phi t} = M + \int_0^t \alpha\, e^{\beta n}\, dn + \frac{td\,(\gamma e^{-\phi t})}{dt}$ -------------- (7)
Using the rules of integration by parts,
$B(t) = \int_0^t \alpha\, e^{\beta n}\,\mathrm{dn} = \frac{\alpha}{\beta}\,(e^{\beta t} - 1)$ --------------------------------- (8)
Similarly, by simple differentiation,
$\frac{d(\gamma e^{-\phi t})}{dt} = \gamma\phi e^{-\phi t}$ ------------------- (9)

Hence, the optimal time solution equation is written as;
$\alpha\, e^{\beta t}\,(t-1/\beta) + \gamma e^{-\phi t}(1+\phi t) + \alpha/\beta = M$ -------------(10)
Equation (10) is a transcendental equation as it contains both exponential and constants, hence solving for t is a complex issue, rather, according to the report of Nahmias (2009), it is more ideal to find the value of t that will make the left-hand terms as close enough to the replacement cost (M), which will define the optimal value of t. At this juncture, a graphical solution may be recommended where a constant value of M is drawn against varying values of the left-hand terms (L H S) of equation (10) for various values of t, until there is an intercept of the lines. The value of t at the point of intersection is the optimal time.

2.4 AN OVERVIEW OF LINEAR REGRESSSION BASED MAINTENANCE AND SALVAGE COSTS FUCTIONS PARAMETERS DETERMINATION

The assumption of linearity in costs data set is the bases for the determination of the costs functions parameters in the economic life cycle modelling of optimal replacement time estimate. In the study of Nahmias, (2009), the values of the parameters (γ and ϕ) in equation (10) were evaluated for the salvage cost function based on the assumption that the replacement cost which is the original equipment cost, depreciate with a constant value for every data point or specified duration, such that the value of the salvage cost at time (t=0) which was designated $S(o)$ was equal to the original equipment price.
Consecutive yearly salvage costs where then determined as
S(1)= (1-$\dot{e}$/100) S(0), S(2) =$(1\text{-}\dot{e}/100)^2$, ………, S(j)=$(1\text{-}\dot{e}/100)^j$ where j is the total number of data points in a planning horizon and $\dot{e}$% is the percentage depreciation value. Here data points mean the time sub- division of the planning horizon. This assumption though allows for

the easy determination of the salvage cost rate functions parameters (α, β), but does not allow for the possibilities of appreciable variance in actual consecutive cycle cost due to randomness introduced by fluctuating cost occasion by the envisaged mid extreme-points scenario of possible high and low costs with no uniform distribution within the planning horizon. Similarly, the ratio of the previous cycle maintenance cost to that of the present was defined as constant value. This again eroded the possibilities of lesser expenditures on maintenance cost in some cycles due to chance quality vehicle parts usage, design to reduce frequent break down, operational proficiency, expertise and trust issues of current equipment operator, higher environmental impact due to seasonality and purpose driven friendlier government policies on one hand and higher maintenance cost in other cycles. In some other literature like, Hartman (2005) and ukwu et al. (2024), where the salvage and maintenance cost functions parameters were utilized recommended regression approach in the determination of the parameters although they also introduced the term S(0) into the regression equation as the salvage value of the equipment at time t=0 .

The regression approach to cost parameter determination proceeds with conversion of the costs exponential function to linear form using a logarithmic transformation, but where negative parameters are determined, the regression approach is truncated, since the trend of decreasing salvage value and increasing maintenance costs is reversed by negative parameters, Nahumias, (2009).

In this study, a logarithmic transformation of costs data set is done for linearity and thereafter, the conventional economic life cycle model is employed to determine the optimal replacement time.

Consider eqn. (6) as an exponential function of the salvage cost rate,

Recall eqn. (6)

$S(t) = \gamma e^{-\phi t}$

Fitting eqn. (6) for multiple data points, (where data points could be weekly, monthly yearly, or such time sub – division of the planning horizon) with regression method gives;

$$ln\, S(t_i) = ln\gamma - \phi t_i - - - - - - - - (3)$$

$$let\ lnS(t_i) = \gamma_i$$

$$ln\gamma = a$$

$$and\ \ t_i = \text{xi} = age\ of\ the\ equipment\ (bus), at\ data\ point\ i$$

$$where\ i = 1, 2, 3, - - - - -, j\ , is\ the\ i^{th} data\ point$$

$$and\ j\ is\ the\ total\ number\ of\ data\ points$$

Hence, equation (3) can be written as;

$$\text{Yi} = a - \phi Xi - - - - - - - - - (12)$$

Where a = the constant that is the estimate of the regression line intercept on the salvage cost axis

ϕ = the estimate of the slope of the regression line.

$Y_{\ i}$ = Represent the estimate of the average value of the salvage cost for a specified time.

Similarly, the fit for equation (5) is given as;

Recall equation (5)

$$C(t_i) = \alpha e^{\beta t}$$

$$lnC(t_i) = ln\alpha + \beta t_i - - - - - - -(13)$$

$$let\ lnC(t_i) = \theta i$$

$$ln\alpha = b$$

$$and\ t_i = T\text{i}$$

Hence, equation (13) can be rewritten as,

$$\theta_i = b + \beta T_i ---------- (14)$$

θ_i= the estimate of the average value of the maintenance cost for a specified cycle time.
The normal equation for equation (12) are;

$$a = \frac{\Sigma Y_i - \phi \, \Sigma \, X_i}{j} ------------- (15)$$

$$and \; \phi \Sigma X^2\text{i} + a\Sigma \, X_i = \Sigma \, X_i Y_i ---------16)$$

The normal equation for equation (14) are;

$$\beta = \frac{J\Sigma T\text{i}\theta\text{i} - (\Sigma T\text{i})(\Sigma\theta\text{i})}{j(\Sigma T^2\text{i}) - (\Sigma T\text{i})^2} ------- (17)$$

$$\text{b} = \frac{\Sigma\theta\text{i}}{J} - \frac{\beta(\Sigma T\text{i})}{J} = \bar{\theta}_i - \beta \, \bar{T} -----------(18)$$

Where $\bar{\theta}$ and $\bar{T}_i$ are mean values

2.5 DETERMINATION OF THE COSTS FUNCTION PARAMETERS BASED ON THE PROPORTIONAL COSTS APPROACH WITH UNTRANSFORMED COSTS DATA

In the proportional cost approach to the determination of the costs functions parameters, the salvage and maintenance cost in each consecutive cycles progresses with a constant proportion. Hence equation (6) at t=0 becomes;

$S(0) = \gamma e^{-\phi 0} ------- (19)$

Where $S(0)$ is the salvage value of the bus before the commencement of the replacement cycles at t=0 and it is equivalent to the bus original purchase price.

Since $S(0)$ = M at t = 0,

Let the depreciation in the salvage value of the bus in percentage be $\dot{e}\%$ of (M), such that the salvage value after one year (S(1)) will be given as ;

$S(1)$ (1-$\dot{e}$/100) M = $\gamma e^{-\phi(1)}$

Since γ= M, we have

$S(1)$ = (1-$\dot{e}$/100)M = M$e^{-\phi}$ ------20)

Hence,

$$e^{-\phi} = (1 - \frac{\dot{e}}{100})$$

$$\Rightarrow \phi = -\text{In}\left(1 - \frac{\dot{e}}{100}\right) ---------(21)$$

Therefore

$S(t)$ = M$e^{-(-In\left(1-\frac{\dot{e}}{100}\right)t}$ ------- (22)

Similarly, if the maintenance cost of the bus for the first year B(1) is given , and the incremental rate is also stated as it is with the conventional approach , let us assume B(1) = # L and the maintenance cost incremental rate per year is c% , then we have that,

$$\frac{C(\text{t})}{C(\text{t}-1)} = 1 + \frac{c}{100} --------- (23)$$

And

$$\frac{\alpha}{\beta}(e^{\beta} - 1) = L \; for \; t = 1 \; year ------(24)$$

Where L is the maintenance cost for the first year.

But, $\frac{\propto e^{\beta t}}{\alpha \, e^{\beta(t-1)}} = e^{\beta} = (1 + C/100) ------ (25)$

Hence, $\beta = In \, (1 + c/100) ---------- (26)$

From equation (8)

$$\alpha = \frac{B(1)\times\beta}{e^{\beta}-1} = \frac{L \times I\,\mathrm{n}(1+ c/100)}{e^{In(1+ c/100)}-1} ------- (27)$$

Hence, equation (5) can be rewritten as;

$$C(t_i) = \frac{L.In\ (1+c/100)}{e^{In(1+c/100)}-1}\ \left[e^{In(1+c/100)t}\right] ------- (28)$$

Let the salvage cost per i^{th} data point be Pi and that of the maintenance cost be Ki , the value of and e can be determined from simple ratio and proportion according to Will, (2004) as follows;

$$\dot{e} = \left[1 - \frac{\sum_{i=0}^{j} \mathrm{p_{(i+1)}/p_i}}{\mathrm{j}-1}\right]100------- (29)$$

Similarly,

$$\mathrm{C} = \left[1 - \frac{\sum_{i=1}^{j} \mathrm{K(i+1)/Ki}}{\mathrm{j}-1}\right]100------- (30)$$

2.6 THE PROPOSED IMPROVEMENT TO THE PROPORTIONAL COSTS APPROACH WITH LOGARITHMIC TRANSFORMATION IN THE DETERMINATION OF THE MAINTENANCE AND SALVAGE COST FUNCTIONS PARAMETERS

In the application of the proportional costs approach, the defined incremental and depreciation values of the maintenance and salvage cost of an equipment (bus) must reflect the variations in each cycle's cost value. This is only achieved, if the variation is equal or with minimal deviations, such that, each consecutive maintenance and salvage cost bears a constant proportion to the previous. Otherwise, the computational error introduced by cost terms with large deviations from the proportional value will distort the functions parameters true values, thereby generating false replacement time. Supposing as envisaged there are closely and widely dispersed costs values within the maintenance and salvage cost distribution, then there is the need to introduce linearity to the costs distribution to reflect a near uniform cost distribution within the replacement cycles.

The adjustment to the proportional cost approach of the conventional optimal equipment time determination, requires the non-linearity in the maintenance or salvage cost as the case may be, to be reduced as much as possible through a logarithmic transformation of the costs to their lowest form. This approach will help reduce disparity of large variance and provide the window to assess a dispersed data set. This one step data transformation procedure supposing a data set is represented by P_i is given as; P_{iT}

$$P_{iT} = \ InP\mathrm{i} \ ------(31)$$

Where P_{iT} is the transformed data set.

When the transformation is concluded, a depreciation value ($\dot{e}$) is determined for the salvage cost as;

$$\dot{e} = \left(1 - \frac{\sum_{i=1}^{j} \frac{Y\mathrm{(I+1)T}}{\mathrm{YiT}}}{j-1}\right) \text{------------ (32)}$$

I=0, 1, 2…... j

The term $\left(1 - \frac{\sum_{i=1}^{j} \frac{\mathrm{Y(I+1)T}}{\mathrm{YiT}}}{j-1}\right)$ is the multiplier of each data points salvage cost to give the succeeding data point salvage cost.

Here, unlike the proportional costs approach where the value of $\dot{e}$ can be determined from the untransformed data set, $\dot{e}$ is carefully selected to minimize its error effect on the optimal time estimation as it relates to the parameter ϕ.

Similarly, the percentage increase in consecutive data points or service ages maintenance cost (c) is determined as;

$$C= \left[1 - \frac{\sum_{i=1}^{j} \frac{\theta(i+1)T}{\theta iT}}{J-1}\right]100 \text{ ------------------- } (33)$$

2.7 DETERMINATION OF COST FUNCTION PARAMETERS WITH THE PROPORTIONAL COST APPROACH WITH UNTRANSFORMED FIELD COSTS DATA SETS

An inaccurate choice of maintenance and salvage costs functions parameters possibly influenced by the non - linearity of costs data set can lead to wrong replacement time optimization, thereby hindering profit maximization from an equipment utilization within its useful life cycle. Below is the actual field record of the maintenance and salvage costs for 10 years operation of a commuter bus in Free will line in Owan East, Edo State Nigeria for optimal replacement time analysis

Table 2.1: Commuter bus operational activities for a period of 10 years of a Toyota HIACE, 2010 Model (Hummer) 15 Seaters.

Original purchase price	Years of operations	Maintena nce-nce Cost C(t) #	Salvage Cost S(t) #	Maintenance Activities	Remarks
4,500,000	0	0	4,500,000	No Servicing	Salvage value was based on potential buyers offer at no increase in replacement cost
4,500,000	2013	105,000	4,000,000	General Servicing	
4,500,000S	2014	216,000	3,900,000	Servicing, brake pads and shock changes	Same Reasons for Salvage Value, but gradual increment in tariffs
4,700,000	2015	310,000	3,750,000	Servicing, tyres Replacement. Purchase of battery, exhaust and shock repairs	Maintenance due to change of drivers, salvage cost due to rising inflation
4,900,000	2014	360.000	3,400,000	Servicing, panel work, Radiator change, electrical work and uncaptured expenses	Better operational factor from new driver causing marginal maintenance cost
5,150,00	2017	402,000	2,950,00	Servicing, shock and brake pad changes rings and piston change replacement of water pump	Weakened engine operation resulted in lower salvage value and less optimal performance

5,300,000	2018	710,000	2,900,000	Engine replacement, servicing, steering bus replacement	Marginal salvage cost decreases due to engine replacement
5,550,000	2019	715,000	2,870,00	Shock replacement servicing, starter and un-captured expenses.	High parts cost due to COVID issues, also resulting in marginal salvage cost decrease because of high bus replacement cost
5,650,000	2020	800,000	2,860,000	Servicing lyres hub and sundry parts replacement	Higher bus replacement cost resulting in marginal decrease in salvage cost and steady rising maintenance cost
5,920,000	2021	980,000	2,800,000	Servicing panel work including respraying and brake pads replacement	High exchange rate and inflation causing marginal salvage value decrease and increase in maintenance cost

6,000,000	2022	1,002,000	2,750,000	Servicing, shocks and brake pad replacement. replacement of top engine block and general maintenance	Same reasons as before for marginal decrease in salvage value and maintenance cost, increasing replacement cost is due to, inflation higher exchange rate and high tariffs

Table 2.2: Transformed and Un-untransformed Data Set of Maintenance and Salvage Costs

DATA POINTS (i)	**InC(t_i) Y_{iT} # 1,000,000**	**In(t_i) θ_{iT} 10,000**	$\frac{Y_{(i+1)T}}{y_{iT}}$	$\frac{\theta_{(i+1)T}}{\theta_{iT}}$	$\frac{p_{(i+1)}}{p_i}$	$\frac{k_{(i+1)}}{k_i}$
0.	1.504	-	-	-	-	0
1.	1.386	2.35	0.92	1.31	0.89	2.06
2.	1.361	3.07	0.98	1.12	0.98	1.44
3.	1.322	3.43	0.97	1.04	0.96	1.16
4.	1.224	3.58	0.93	1.03	0.91	1.12
5.	1.082	3.69	0.88	1.15	0.87	1.77
6.	1.065	4.26	0.98	1.00	0.98	1.01
7.	1.054	4.27	0.99	1.03	0.99	1.12
8.	1.051	4.38	1.00	1.05	1.00	1.23
9.	1.030	4.59	0.98	1.00	0.98	1.023
10.	1.012	4.61	0.98	-	0.98	
Σ	13.091	38.23	9.61	9.73	9.54	11.93

To determine the values of e and c, for the untransformed data set, recall equation (29) and (30)
Recall equation (29);

$$\dot{e} = 100\left[1 - \frac{\Sigma_{i=1}^{j}\frac{p_{(i+1)}}{p_i}}{j-1}\right]$$

For i = 0, 1, 2…j
From the entries of $p_{(i+1)}$ and p_i in table 2.3, we have;

$$\dot{e} = \left(1 - \frac{9.54}{10}\right)100 = (0.056)100 = 5\%$$

Recall equation 21
$\phi = -\text{In}\left(1 - \frac{\dot{e}}{100}\right)$
$\Rightarrow \phi = -\text{In}\ 0.95 = 0.0513$
Hence S(t_i) from equation 22 becomes;
S(t_i) = 4,500,000 $e^{-0.0513t_i}$
Similarly, using equation (30)

$$C = \left(1 - \frac{\Sigma_{i=1}^{j}\frac{K_{(i+1)}}{k_i}}{j-1}\right)100$$

For i = 1, 2, 3…j
From the entries of $K_{(i+1)}$ and k_i in table 4.3 we have that,

c = $\left(1-\frac{11.93}{9}\right) X\,100 = (.33)100$
= 33%

To determine the maintenance cost for this value of c,
Recall equation (25)
e^{β}= $(1+\ \text{c}/100)\ =\ ($ 1+ 33/100) = 1.33
$\beta\ =\ $ In 1.33 $\ = 0.285$
Given that L = # 105, 000 = B(1)
From equation (27)
$\propto = \frac{B_{(i)}\,\beta}{e^{\beta}-1} = \frac{105\ 000 \times 0.285}{e^{0.285}-1} = \frac{29{,}925}{0.329762020}$
$\propto =\ 90,747.26$
Hence $\propto/\beta$ = 3, 184, 114. 334
⇒$B(t_i)$ = 3, 184, 114. 334 $(e^{0.285t_i} -\ 1)$

$B(i)$ = # 1,050,000.001 as against # 105, 000 which is 10,000% increment
For the salvage value estimation under this untransformed data approach,
Recall eqn. (22)

$B(t_i)$ = M $e^{-0.0513t_i}$
= 4,500,000 $e^{-0.0513ti}$
For t_i = 1yr
$S(t_i)$ = 4, 500, 000 $e^{-0.0513}$
= # 4, 274, 971.33 as against # 4, 000, 000 in table 4.1
Diff = #274,971.33 for the salvage cost
The optimal time replacement equation for these new sets of parameters from equation (10) becomes;

$$90{,}747.26e^{\mathbf{0.285}t_i}(t_i-\frac{1}{\beta})+4{,}500{,}000e^{-\mathbf{0.0513}t_i}(1+0.0513t_i)+3{,}141{,}114.334\ =M$$

2.8 DETERMINATION OF THE OPTIMAL REPLACEMENT TIME AND THE COSTS' FUNCTION PARAMETERS FOR THE IMPROVED PROPORTIONAL COSTS APPROACH.

The conventional method as it is referred to is in this study, allows the salvage cost at time (t=0) to be set equal to the original equipment (bus) purchase price. As earlier said, it sets a uniform variation for the equipment ages maintenance and salvage cost respectively, which presupposes uniform distribution of the respective data sets.
The proportional increase in consecutive data point or ages maintenance costs (c) is not chosen arbitrarily, and to determine its value by the adjustment to the conventional method recall equation (32)

$$c=\left(1-\frac{\Sigma\frac{\boldsymbol{\theta}_{(\boldsymbol{i}+\mathbf{1})\boldsymbol{T}}}{\boldsymbol{\theta}_{\boldsymbol{iT}}}}{j-1}\right)100$$

From the entries in Table 2.3, $\frac{\Sigma_{t=1}^{j}\frac{\boldsymbol{\theta}_{(\boldsymbol{i}+\mathbf{1})\boldsymbol{T}}}{\boldsymbol{\theta}_{\boldsymbol{iT}}}}{j-1} = \frac{9.73}{9} = 1.08$
⇒ c= $(1 - 1.08\ )100$ = 8%

Similarly for the determination of the depreciation rate ($\dot{e}$), we proceed as follows;
Recall equation (31),

$$\dot{e} = \left[1 - \left(\frac{\Sigma \frac{Y_{(i+1)T}}{Y_{iT}}}{J-1} \right)\right] 1OO$$

But $\frac{\Sigma \frac{Y_{(i+1)T}}{Y_{iT}}}{J-1} = \frac{9.61}{10}$

$\dot{e}$ = $(\ 1 - 0.96 \ )100 = (\ 0.04 \ )100$

$\dot{e}$ = 4%

The transformation of the maintenance costs has reduced the disparity to a mean value of 3.82 and a mean deviation of 1.67 for the smallest term and 0.79 for the highest number.
The proportional increase in consecutive ages cost is about 8% for the maintenance cost and 4% depreciation value for the salvage cost analysis. From the above analysis of the value c = 8 % and e $=$ 4%, β and α can be determined as follows;
For the determination of the maintenance cost function, recall equation (23),

$$\frac{C_t}{C_{(t_i-1)}} = 1 + c/100 = 1 + \frac{8}{100} = 1.08$$

- From equation (25),

$e^{\beta} = (1 + c/100)\left(1 + \frac{8}{100}\right) = 1.08$

$\Rightarrow \beta = \text{In}\ (1.08) = 0.0769 \approx 0.0769$

Given that L = # 105, 000

- then from equation (27) we have,

$$\propto = \frac{B(i).\beta}{e^{0.077} - 1}$$

$\propto$ = 101, 009

But $\propto/\beta = \frac{101{,}009}{0.077}$ =1, 311, 810.05

$\Rightarrow C(t_i) = 101{,}009\ e^{0.7077 t_i}$

And $B(t_i)$ = 1, 311, 810.

TABLE 2.3 COSTS AND OPTIMAL REPLACEMENT TIME COMPARISON FOR THE TRANSFORMED AND UNTRANSFORMED DATA

DATA POINTS	ACTUAL YEARLY SALVAGE COST A (#)	ESTIMATED YEARLY SALVAGE COST FOR TRANSFORMED DATA B (#)	ESTIMATED YEARLY SALVAGE COST FOR UNTRANSFORMED DATA C (#)	B – A (#)	C – A	ACTUAL YEARLY MAINTENANCE COST (#)	ESTIMATED MAINTENANCE COST FOR TRANSFORMED DATA E (#)	ESTIMATED MAINTENANCE COST FOR TRANSFORMED DATA F (#)	E-D (#)	F –D (#)	OPTIMAL REPLACEMENT TIME FOR TRANSFORMED DATA (YEAR)	OPTIMAL REPLACEMENT TIME FOR UNTRANSFORMED DATA (YEAR)
2013	4, 000, 000	4,319,231.085	4, 274, 971.33	319, 231.09	274,971.3	105,000	105,000	1,050,000.001	0	945,000	1-5	∞
2014	3, 900, 000	4, 145, 723.81	4, 061, 195. 5	245, 723.81	161,195.5	216,000	218,404.42	2,446,250.13	-2,404.42	2,230,250		
2015	3, 750, 000	3, 979, 186.48	3, 858,109.90	229,186.42	108,1,09.9	710,000	340,886	4,302,930.54	60,354	3,992,930.5		
2016	3, 400, 000	3, 819, 339.10	3, 665. 179. 8	419,339.1	265,179.8	1,002,000	473,171.201	6,771,873.64	521,393.03	6,411,873.6		

2017	2, 950. 000	3, 665, 912. 24	3, 481, 897. 50	715,912.24	531,897.5	310.000	616,044.87	10,054,980.42	30,886	9,652.980.4
2018	2, 900, 000	3, 518, 650	3, 307, 780. 42	618,650	407,780.42	360,000	770,345.30	14,420,731.16	113,171	13,710,731.2
2019	2, 870, 000	3, 377, 302. 80	3, 142, 370. 03	507,302.8	272,370.03	402,000	937,015.05	20,226,140.71	214,044.9	19,511,141
2020	2, 860, 000	3, 241, 633. 59	2, 985, 231. 80	381,633.59	125,231.8	715,000	!,117,015.68	27,945,954	222,015.05	27,145,954
2021	2, 800, 000	3, 111, 414. 40	2, 835, 951. 2	311,414.4	35,951.2	800,000	1,311,424	38,211,468.33	317,015.70	37,231,468.3
2022	2, 750, 000	2, 986, 426. 13	2, 694, 135. 56	236,426.13	-55,864.4	980,000	1521,393.03	51,862,160	331,424	50,860,160

3. RESULT AND DISCUSSION

3.1 RESULT

One of the conditions for the application of the regression (least squares) method to variable maintenance and salvage cost estimation is that, the functions parameters must be greater than zero i.e. β and $\phi > 0$, Nahmias, (2009). This condition was not met in the application in this study, as the estimated regression line slope was negative i.e. $\phi = -0.0077$. Hence, the regression method was not the best approach in the cost parameter determination. This would have resulted in rising salvage costs if the computation with a negative ϕ was done, negating the normal trend of depreciation in equipment (bus) value with age. Although, this negative value negates the normal salvage value trend, but reports a unique situation where, increase in replacement cost of an equipment (bus), discourages replacement to encourage maintenance, thereby resulting in either slow depreciation as reported in table 2.1 or a gradual increase in the operational equipment (bus) value due to high demand for fairly used equipment, which in some instance could lead to increase in salvage value of used equipment.

The result of the logarithmic transformation reported in table 2.2 of the maintenance and salvage cost data sets reduced the disparity in their data point cost variation and allowed for a more representative choice of incremental rate and depreciation value for the maintenance and salvage cost respectively. Hence, in the application of the modified logarithmic based cost parameter determination for the conventional method, results in table 2.3 were more reflective of the entries in table 2.1 for the actual cost values. The disparity in the estimated cost values reported in table 2.3 for the maintenance cost was due to the wide variance in the actual consecutive data points values, which the transformation sought to reduce but obviously to an extent. This is supported by the unrealistic and irreconcilable differences between the estimated values of the untransformed data set in table 2.3 and the true value of table 2.1, which reported 1000% increase in the first-year maintenance cost. For the salvage cost, the entries in table 2.1 showed that the depreciation of the bus between time zero (t=0) to t=1 yr. was about 11% against 2% depreciation in other consecutive data points cost from t=1yr onward. This disproportionate depreciation was heightened in the estimated values in table 2.3, since the original purchase price is the bases of the salvage cost function.

The optimal replacement time of 1.5yrs or 18months seems to be reasonable for a replacement cost of N4, 500,000 form table 2.1, since at the optimal time, the bus can still sell for N4, 500,000 at an estimated operational cost of N429, 022.57.

In the case of the untransformed costs formulated optimal replacement time procedure, the estimated optimal time is not achievable, since at 0 yr., the control value (LHS) is 7,323, 498.92 which is about 1.6 of M, hence a range of optimal time does not exist for the untransformed data for t.

The sensitivity test which involved comparing results of the transformed data sets to the untransformed sets showed that, widely dispersed data will result to a sub-optimal replacement time as the result of the untransformed data produced a replacement time that does not feasible.

3.2 CONCLUSION

The conclusion drawn at the end of this study is that, while utilizing the conventional economic life cycle salvage and maintenance costs analysis in the optimal equipment replacement time determination, the costs data set must be transformed to introduce linearity in the presence of obvious wide range disparity as it is with the maintenance cost. One of such normalization procedures is the logarithmic transformation as employed in the cause of this study.

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